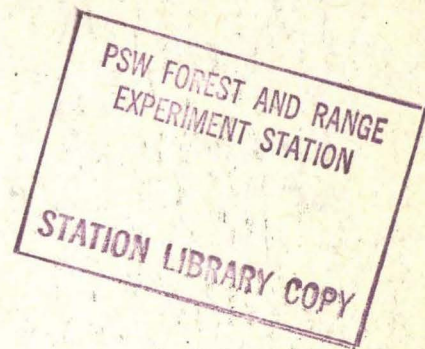




OPTIMAL SCHEDULES IN KILN-DRYING
THROUGH DYNAMIC PROGRAMMING

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by G. Rensi and A. Weintraub

1. Introduction

The purpose of drying wood in kilns is to reduce the moisture content in the wood to a desired level i) efficiently, ii) without impairing the quality of the wood material. Drying time, cost, and quality change are the three fundamental aspects of drying efficiency. The shorter the drying time, the faster the production of wood per unit of time. Drying cost per unit of wood dried is, by and large, inversely related to drying time. The quality of the dried wood can be expressed in terms of the average final moisture content as well as other quality parameters such as shrinkage, collapse, and others. The loss in quality incurred through drying can be called 'drying degrade'.

A drying schedule is a predetermined program of temperature, humidity, and air velocity inside the drying-kiln. The acceptability or desirability of a drying schedule can be defined in terms of its drying efficiency.

This paper illustrates the application of dynamic pro-

gramming to the problem of efficient kiln drying. An analytical representation of the drying process is postulated, whereby wet wood of given characteristics is placed into a kiln of given characteristics for a controlled amount of time and for a controlled time sequence of values of temperature and wet-bulb depression in the kiln atmosphere. The length of the drying period and the values of temperature and wet-bulb depression at each point of such period will be the decision variables in this process. Air velocity will be taken as a fixed parameter in the process.

The main characteristics that describe the state of the wood at each moment are its moisture content and a measure of deformation of the wood from its desired basic form. For many cases these two state variables will suffice to define the process with adequate approximation.

We assume that the wood species and type of material within the kiln are quite homogeneous in a given drying run, so that we can approximate the condition of the wood in the kiln by representative state values. This implies a certain uniformity in the response of all pieces of wood.

The problem will consist then in finding a time-sequence of values of temperature and depression to be applied to the kiln, until the wood is dried to an acceptable level of moisture, while maintaining predetermined standards of qual-

ity. The mentioned controls will usually be set daily and remain constant during that interval. The decisions will be based on optimization and constraining factors.

1. Direct costs: There will be a variable cost associated in each period to fixing the control variables. This cost will depend on the values of the controls of the previous and present time interval and the moisture content of the wood at the start of the present interval. There will also be a fixed cost per day due to overhead, maintenance, inventory holding, depreciation, etc.

2. Indirect costs: At the termination of the process, a fraction of the pieces will have to be rejected because of falling outside the range of acceptable moisture content. This factor corresponds to a variance in the distribution of the moisture content of the wood in the kiln.

3. Constraints: Care has to be taken in the determination of the sequence of decisions. Too fast drying will produce stresses in the wood leading to unacceptable deformation in the pieces. Thus, for a given state of the wood and kiln, certain values of the control variables will not be permissible.

The decision process is divided into two components:

- a) Quantitative characterization of the drying process.
- b) Determination of an efficient schedule of the drying process in the kiln.

The functions describing the drying process were determined empirically, through experimentally observed values and statistical regression tools. No in depth analysis or consideration was given to the engineering characteristics of the drying equipment nor to the physico-chemical aspects of the raw materials and their changes in the course of the drying process. Part a) is detailed in section 2.

The functions that were determined to characterize the drying process turned out to be complex, and noncontinuous for the case of costs. For this reason, a solution approach based on optimal control or nonlinear programming is not adequate. A dynamic programming approach is thus proposed, which can handle the complexities, and particularly the non-continuity of the cost function, by considering only discrete values of states and decisions. Part b) is developed in section 3.

2. Quantitative characterization of the drying process

Notation:

N = drying period (number of days)

t = drying interval (day). Note that

$$t = 1, 2, 3, \dots, N$$

M_t = moisture content in the wood at the beginning
of drying interval t

T_t = temperature of kiln atmosphere during time
interval t

D_t = wet-bulb depression during time interval t

Note: The subscript t is used to indicate both a) the value of state variables at the beginning of a discrete time interval, and b) the (constant) value of the control variables throughout a discrete interval.

2.1 The Drying-Rate Equation

The drying rate equation expresses the reduction of moisture content in the wood per unit of time. In terms of the analytical factors affecting the drying process the rate of drying is directly related to temperature and inversely related to humidity in the kiln atmosphere. Using wet-bulb

depression as a proxy variable for humidity (the higher the wet-bulb depression, the lower the humidity), the drying rate is directly related to wet-bulb depression.

According to Bramhall's (1975) formulation, wood drying is most conveniently expressed by the following equation

$$R(M) \, dM = P(DT) \, dt$$

where M is moisture content, DT is the combined temperature and wet-bulb depression variable. Note that

$$R(M) = Ae^{-bm}$$

is the "resistance to flow" or resistance to the flow of moisture to the outside in the drying process, and

$$P(DT)$$

is the vapor pressure differential associated with temperature-depression DT . For the kiln-drying situation considered in this discussion, the resistance to flow equation fitted to the data was

$$R(M) = 112.0 \, e^{-0.044}$$

The vapor pressure differential value corresponding to DT can be read off Table 1 in the Appendix, section 1. Note also that dt is measured in 24 hr. units. Therefore, if one wants to dry the wood from M_1 to M_2 in 24 hrs., one will have to provide a vapor pressure differential equal to

$$\begin{aligned} P_0 &= \int_{M_1}^{M_2} R \, dM = b^{-1} A \left[e^{-bM} \right]_{M_1}^{M_2} \\ &= 2545.5 (e^{-0.044 M_2} - e^{-0.044 M_1}) \end{aligned}$$

Given the required vapor pressure differential P_0 , there exists a set of values of DT such that $P(DT) = P_0$. For additional details on the drying rate equation, see Appendix A. Given the moisture content at the beginning of interval t , the moisture content at $t+1$ (i.e. after 24 hrs.) is

$$M_{t+1} = - \frac{1}{0.044} \ln(e^{-0.044 M_t} + \frac{1}{2545.5} P(DT_t))$$

where DT_t is the temperature-depression setting for the time interval t .

2.2 Drying Degrade Equations and Loss Functions

Kiln conditions created by high dry-bulb and low wet-bulb temperatures can be very damaging to lumber if they are maintained for too long a period of time. The higher the wet-bulb depression (difference between dry-bulb and wet-bulb temperatures) the lower is the moisture content at the wood surface. The consequent high differential between surface and core moisture contents tends to result in shrinkage based defects. Some species are subject to collapse type defects if they are excessively heated while the moisture content is near complete saturation, i.e., toward the beginning of the drying period.

Two major types of defects have been quantitatively characterized, namely, a) moisture content defects,

b) shrinkage based defects. A measure of quality loss due to inappropriate final moisture content has been formulated on the basis of i) the average moisture content at the end of the final drying interval, ii) the average moisture content at the beginning of the final drying interval, iii) the maximum acceptable moisture content in the dried wood. Let Q represent the fraction of wood pieces with moisture content exceeding the tolerance level, M_* . M_* represents the upper bound of B_M , the acceptable range of final moisture content values. At the end of drying interval N , Q can be computed as a function of M_N , M_{N+1} , where M_{N+1} is the value of the moisture content at the end of interval N .

$$Q = Q(M_N, M_{N+1})$$

Details on the computation of Q can be found in the Appendix, section 2.a. One can now compute the loss attributable to excess moisture content, i.e., $L(Q)$

$$L(Q) = V \cdot (1-s) \cdot Q$$

where V is the value of a defect-free kiln load and s is the ratio of the salvage value over the normal value of the dried wood per unit.

In order to account for the drying degrade due to shrinkage based defects, a proxy variable representing residual stress level has been selected. A semi-logarithmic equation relating residual stress level to wet-bulb depression in each

time interval has been postulated. On the basis of the estimated residual stress, a function R , measuring the fraction of wood pieces with shrinkage defects, can be evaluated. A maximum value of R , i.e., R^* , is defined to provide a bound on the admissible levels of wet-bulb depression. For further details, consult the Appendix, section 2.b.

2.3 Drying Cost

The drying cost can be calculated on a per period (24 hrs.) basis. Let C represent the cost of drying the wood during the time period t . C can be expressed as the sum of two components, i.e., fuel and non-fuel costs.

$$C = F \cdot C_F + C_H$$

F is the amount of fuel in BTU's required to heat the kiln. C_F is the unit cost per BTU of fuel. C_H is the fixed cost per day. C_H includes overhead cost, depreciation, maintenance and repair cost (fixed), inventory in process cost and electricity cost. These are fixed costs that occur independently of the type of kiln schedule adopted.

The fuel consumption, F , can be calculated as the sum of four components, namely,

- F_1 the BTU equivalent required to evaporate
 the water out of the wood
- F_2 the BTU equivalent required to increase

the temperature inside the wood

F_3 the BTU equivalent required to increase the temperature in the kiln atmosphere

F_4 the BTU equivalent required to heat the air taken into the kiln when reducing the humidity in the kiln (through air exchange).

F_1 is a function of ΔM_t , the change in wood moisture content, and T_t , the temperature in the kiln atmosphere. W , the dry weight of the wood load is an important parameter of the function. For details, see Appendix, section 3.a.

F_2 is a function of temperature and depression in the kiln atmosphere both in drying interval t and in the preceding drying interval $t-1$. More detail on F_2 is given in Appendix, section 3.b.

F_3 is a function of temperature and depression in the preceding drying interval $t-1$, as well as temperature in interval t . For more detail, see Appendix, section 3.c.

F_4 is a function of temperature and depression both during the drying interval t and the interval $t-1$. The dry weight of the wood W and the temperature and depression of the atmosphere outside the kiln, T_0 , D_0 , are also important parameters of the function. For more detail, see Appendix, section 3.d.

Using the symbol DT_t to denote temperature-depression

pairs in interval t , the fuel consumption function F can be represented as

$$F(DT_{t-1}, DT_t, M_t) = F_1(M_t, DT_t) + F_2(DT_{t-1}, DT_t) \\ + F_3(DT_{t-1}, DT_t) + F_4(DT_{t-1}, T_t)$$

Hence the drying cost equation can be formulated as

$$C_t(M_t, DT_{t-1}, DT_t) = C_H + C_F F(M_t, DT_{t-1}, DT_t)$$

3. Dynamic Programming Model

We present now a dynamic programming formulation to deal with the optimization problem described in section 1.

3.1 For each day, or interval t , there will be two decision or control variables defined: the temperature T_t and depression D_t to be applied in the kiln on that day. In addition, for each interval, we will define three state parameters: the moisture content M_t at the beginning of period t , and the two decision variables T_{t-1} , D_{t-1} of the previous period, which play a role in the costs involved in period t : $C_t(D_t, T_t, D_{t-1}, T_{t-1}, M_t)$.

For the purposes of this analysis, the two decision variables can be combined into one, DT_t , which will reflect a joint level of temperature and depression. In this form, the day by day process will be controlled by one decision variable and two state parameters. The cost function is redefined now to be in period t as $C_t(DT_{t-1}, DT_t, M_t)$.

The noncontinuous nature of the cost function, as shown in section 2, inhibits the use of tools of optimal control. We establish a dynamic programming procedure which enables to optimize the approximated drying process through the control of the day by day operations. This implies that a discrete approximation of the original problem will have

to be solved, in which temperatures, depressions and moisture contents will be assigned discrete values at determined intervals. Thus, the levels of temperature, for example, will be defined only for those discrete values, as a set (T_t^j) . This approximation, done properly, does not really imply a limitation to the approach, as the decision mechanisms, such as temperature settings, for example, are geared towards discrete values (intervals of two degrees for example). This consideration is due to the fact that at some point it makes no sense to determine finer levels of decision, as the setting and control of the variable is not that exact. The same type of argument permits to define discrete intervals of time, since settings are not varied continuously but at given intervals of time.

At the start of an interval t during the drying process, a present moisture content M_t and the decision variable DT_{t-1} corresponding to the previous period are known. A decision must then be made on the control variable for period t , DT_t . This decision leads to three immediate consequences: a cost associated to this decision (function of M_t , DT_{t-1} , and DT_t), a resulting moisture content M_{t+1} at the end of period t , and a level of residual stress which may cause deformation of the wood pieces. As has been shown in section 2,

given M_t , a control variable DT_t may be unfeasible due to excessive deformation.

For certain values of M_t , the process may be allowed to terminate if the existing moisture content is within the acceptable bounds of final moisture content. If that is the case, a loss must be measured attributed to the variance in distribution of the moisture among the slats, which is a function of M_t and DT_t .

We note that there usually will be more than one control variable in period $(t-1)$ that leads to any given moisture M_t^j . This can come about in two forms:

i) Starting from different moisture contents M_{t-1}^j and M_{t-1}^k , different controls may lead the moisture content to a same value M_t^m . ii) Starting from the same moisture content in period $(t-1)$, M_{t-1}^j , two different controls DT_{t-1}^r and DT_{t-1}^s could lead the moisture content again to a same value M_t^v . This fact is due to the contraction of the two real controls into one decision variable for analytical purposes.

We can describe the problem as that of determining a schedule of controls (DT_t^j) that leads from a starting moisture content M_0 to an acceptable moisture content at the end of N periods, N being undetermined, while maintaining an acceptable average deformation and with a minimum total cost. The total cost is given by the following expression:

$$Z = \sum_{t=1}^N C_t(DT_{t-1}, M_t, DT_t) + L(M_N, DT_N).$$

Prior to introducing the dynamic programming algorithm to solve this problem, we must develop some results.

Consider a moisture content M_t^j at the start of period t and a control variable DT_t^r . Let $I_{t-1,j}$ be the set of all control variables of period $(t-1)$ which can lead to that value M_t^j from some starting moisture content in period $(t-1)$. Then, to each element i in $I_{t-1,j}$ there will correspond a starting moisture content M_{t-1}^s in period $(t-1)$. We can calculate the total cost involved, which is composed of the cost of reaching M_t^j , when control i was used in period $(t-1)$, plus the cost of using controls DT_t^r in period t . The latter cost is a function of M_t^j , DT_{t-1}^i , and DT_t^r .

Lemma 1. Let C_t^{ij} be the optimal cost of reaching a moisture content j at the end of period $(t-1)$, when using control variable i in $I_{t-1,j}$ in period $(t-1)$. Let i_0 be the element in $I_{t-1,j}$ that for a control variable DT_t^r in period t minimizes the expression:

$$C_i = C_t^{ij} + C_t(DT_{t-1}^i, M_t^j, DT_t^r) \quad (1)$$

Then, for the given pair (M_t^j, DT_t^r) , $DT_{t-1}^{i_0}$ is an optimal control variable for period $(t-1)$.

Proof:

The objective function of the total schedule can be decomposed as follows:

$$\begin{aligned}
 Z = & \sum_{t'=1}^{t-2} C_{t'}(DT_{t'-1}, M_{t'}, DT_{t'}) + C_{t-1}(DT_{t-2}, M_{t-1}, DT_{t-1}) \\
 & + C_t(DT_{t-1}, M_t, DT_t) + \sum_{t'=t+1}^N C_{t'}(DT_{t'-1}, M_{t'}, DT_{t'}) \\
 & + L(DT_N, M_N).
 \end{aligned}$$

Consider all schedules that reach a moisture content M_t^j at the end of period $(t-1)$. To each of these schedules corresponds a moisture content M_{t-1}^i at the beginning of period $(t-1)$ and an optimal cost C_t^{ij} up to the end of period $(t-1)$. If the control variable DT_t^r is fixed for period t , the last two terms in the summation of Z become fixed. The cost of reaching M_t^j given that DT_t^r is used in period t is given by the first three terms of the summation of Z and is equivalent to:

$$C_t^{ij} + C_t(DT_{t-1}^i, M_t^j, DT_t^r) \text{ for each schedule.}$$

Since by hypotheses,

$$C_t^{i_0 j} + C_t(DT_{t-1}^{i_0}, M_t^j, DT_t^r) \leq C_t^{ij} + C_t(DT_{t-1}^i, M_t^j, DT_t^r) \quad (1)$$

for all i in $I_{t-1,j}$, the optimal schedule that reaches the moisture content M_t^j and uses control DT_t^r , must choose control

i_0 in $I_{t-1,j}$ for period $(t-1)$. This proves the Lemma.

The implication of Lemma 1 is the following: given a determined moisture content at the start of period t , then, at the end of that period, whatever the choice was for the control variable for period $(t-1)$, it has no effects on the optimal decisions of future periods, $t+1, t+2, \dots$

3.2 Flow Chart of the Dynamic Programming Algorithm.

For any moisture content M_j , let $A(M_j)$ be the set of values of the control variable that lead to acceptable average deformations, as defined in section 2.

Flow Chart.

0. Start period 1 with the initial moisture content (m.c.) M_0 and control values DT_0 . Consider each control variable $DT_1^j \in A(M_0)$ that leads to an acceptable average deformation. For this set evaluate the resulting moisture content M_1^j and the optimal costs associated to each control, C_{ij}^1 . Define $TC_0 = \text{optimal total cost} = \text{infinite}$.

1. For a regular iteration t , we have, for every defined moisture content M_t^j , a set of controls that led to that m.c. and an optimal cost associated to each one, C_{ij}^t . Thus, to each defined moisture content corresponds a set of pairs (DT_{t-1}^i, C_{ij}^t) . The first value in each pair refers to one control decision in period $(t-1)$ that led to

the moisture content M_t^j , while the second value refers to the optimal cost associated to reaching the m.c. M_t^j when that control variable was used in period $(t-1)$.

If some m.c. has not yet been feasibly reached in period t , i.e., it is not possible to reach that level of moisture content in t periods within the constraints of the system, the set of pairs (DT_{t-1}^i, C_{ij}^t) is empty and M_t^j is said to be not defined.

For each defined m.c. M_t^j :

For each acceptable control variable $DT_t^r \in A(M_t^j)$, determine

- a) the optimal control variable i_0 in $I_{t-1,j}$ for period $(t-1)$ corresponding to the pair (M_t^j, DT_t^r) , through the recursive relation (1).
- b) the m.c. M_{t+1}^j resulting from the pair (M_t^j, DT_t^r) . Let it be M_{t+1}^m .

From the determination of i_0 , evaluate the optimal cost of reaching state M_{t+1}^m when control variable DT_t^r was used in period t . This defines one entry for the m.c. M_{t+1}^m as (DT_t^r, C_{rm}^{t+1}) .

2. Determine if termination is possible.

- a) Consider all m.c. M_{t+1}^j in the interval of acceptable final moisture content: B_M . If no defined M_{t+1}^j exists in B_M , take $t = t+1$ and go to 1. Otherwise, for every m.c. M_{t+1}^j defined in B_M :

For all controls DT_t^r in period t that led to the m.c. M_{t+1}^j consider:

- i) The optimal cost C_{rj}^{t+1} associated to reaching the m.c. M_{t+1}^j through control DT_t^r .

ii) A loss function due to the variance in the distribution on m.c. in the wood. This loss function was defined in section 2 as

$$L_{t+1} = L(M_{t+1}^j, M_t) \quad \text{or}$$

$$L_{t+1}^{r,j} = L(M_{t+1}^j, DT_t^r).$$

Let $TC(M_{t+1}^j) = \min_r (C_{rj}^{t+1} + L_{t+1}^{r,j})$ be the optimal total cost associated with the final m.c. M_{t+1}^j .

If $TC(M_{t+1}^j) < TC_0$, take $TC_0 = TC(M_{t+1}^j)$ and $DR_0 = L_{t+1}^{r,j}$.

$$b) \quad \text{Let } DR = \min_{r,j} L_{t+1}^{r,j}$$

be the minimum cost due to the loss function possible.

For every m.c. $M_t^j \notin B_M$, check

$$\min_r \left\{ C_{rj}^{t+1} - (DR_0 - DR) \right\} \geq TC_0 \quad (2)$$

If (2) is satisfied for every m.c. $M_{t+1}^j \notin B_M$, there is no possibility that any drying path going through m.c. M_{t+1}^j is optimal, as any savings due to a smaller loss function will not be sufficient to obtain a value better than the incumbent TC_0 . Then, if (2) is satisfied for all m.c. $M_{t+1}^j \notin B_M$, the current solution constitutes an optimal schedule and the process is terminated. Otherwise, eliminate from further consideration all $M_{t+1}^j \notin B_M$ such that (2) is satisfied, take $t = t+1$ and go to 1.

4. Discussion Points .

General applicability of the procedure.

Originally this procedure was formulated in order to calculate optimum schedules in drying pencil slats as an alternative to Claxton's (1968) method. In Claxton's model, the drying behavior of a slat load is extrapolated from experimental observations on samples of slats. In our procedure, the drying rate equation was computed on the basis of observations of the reduction in moisture content for the whole slat load in the kiln. While the equation parameters and numerical examples in this paper relate mainly to the drying of pencil slats, the procedure can also be applied to the case of other wood materials. In order to do so, equations and estimates must be revised to fit the new situation. New drying rate equations, cost and loss functions need to be fitted whenever different equipment, technology and cost-price conditions are present.

Generality of determination of kiln-drying behavior.

The drying rate equation represents only an approximate formulation of the drying process. A more exact formulation would take into account not only temperature, depression and air circulation (the latter treated as a constant para-

meter in our exercise) but also the moisture gradient in the wood pieces. The moisture gradient inside the wood depends on past values of temperature, wet-bulb depression and velocity of air circulation. On the other hand, as confirmed by the findings of Bramhall (1975), the approximation used in this procedure can be usefully applied.

The degree of approximation in the drying rate equation and, consequently, the validity of this procedure will be greater when the optimum schedule will be found within a restricted neighborhood of the schedules presently applied. When working with empirical equations, it is recommendable to work with values within the range of variation of the observations from which the equations were fitted rather than with large extrapolations from such range.

The 'shrinkage defects' equation (see Appendix, section 2.2) has been constructed on the basis of weak empirical as well as theoretical evidence. Very little is known about the behavior of deformation under severe temperature and depression conditions. Prior to applying the procedure here outlined in an operational context, the 'shrinkage defects' equation should be carefully tested and calibrated to reflect both the drying equipment and the type of material processed in the target situation.

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APPENDIX

Section 1. The drying rate equation.

a. Bramhall's revised flux equation

$$R(M) \, dM = P(DT) \, dt$$

R = resistance to flow

P = vapor pressure differential

M = moisture content

DT = combined temperature and wet-bulb depression
variable

b. Bramhall's resistance to flow equation

$$R(M) = Ae^{-bM}$$

Values of A and b were obtained through least-squares
curve fitting on observations from seven drying
experiments with pencil-slats:

$$A = 112.0$$

$$b = 0.044$$

c. Vapor pressure differential calculation from Table 1.

Given a pair T, D, i.e., dry-bulb temperature
and wet-bulb depression, read the values of the
vapor pressure $v(T)$ and $v(T-D)$ from Table 1 and
take the difference. The difference

TABLE 1. Vapor Pressure of Water (mb)

Temp. °F	0	1	2	3	4	5	6	7	8	9
100	66	68	70	72	74	76	78	81	83	85
110	88	91	93	96	99	102	104	107	110	114
120	117	120	123	127	130	134	138	141	145	149
130	153	158	162	166	171	175	180	184	189	194
140	199	204	210	215	221	226	232	238	244	250
150	257	263	269	276	283	290	297	304	312	319
160	327	335	343	351	360	368	377	386	395	404
170	413	423	433	443	453	463	474	485	496	507
180	518	530	541	554	566	578	591	604	617	630
190	644	658	672	687	701	716	731	747	763	779
200	795	812	828	846	863	881	899	917	936	955
210	974	994	1014	1034	1054	1075	1097	1118	1140	1163
220	1185	1208	1232	1256	1280	1304	1329	1355	1380	1407
230	1433	1460	1487	1515	1543	1572	1601	1631	1661	1691

a) Taken from Bramhall (1975), p. 26

$$P = v(T) - v(T - D)$$

is the vapor pressure differential. For example, if $T = 180$ and $D = 50$

$$P = v(180) - v(130) = 518 - 153 = 365$$

Section 2. Quality equations and loss function.

2.1 Final moisture content

The final moisture content distribution among wood pieces can be assumed to be lognormal.

Let m represent the natural logarithm of the final moisture content in the wood pieces. Let $E(m)$, $G(m)$ represent the mean and standard deviation of m . Empirical estimates of $E(m)$, $G(m)$ have been obtained as follows:

$$E(m) = (1-k) \cdot \ln M_{N+1}$$

where k is a small positive adjustment factor, and M_{N+1} is the average final moisture content; and

$$G(m) = .10 E(m) \cdot (1+\Delta)$$

where Δ is the rate of change of the moisture content in the final interval, e.g.,

$$(M_N - M_{N+1}) / 0.5 \cdot (M_N + M_{N+1})$$

On the basis of our assumptions about the final moisture content distribution, the variable z

$$z = z(m) = (m - E(m)) / G(m)$$

is a standard normal variable.

Let M_* be the critical value for moisture content. If a piece of wood has final moisture content $M > M_*$, it is rejected as substandard.

Let $m_* = \ln M_*$ and $z_* = (m_* - E(m)) / G(m)$, then $Q = 1 - \Phi(z_*)$ is the probability of pieces with excessive moisture content. $\Phi(z)$ is the standard normal cumulative distribution function.

The following approximation to $\Phi(z)$ can be used:

$$\Phi(z) \doteq 0.5 + 0.5(1 - e^{-2z^2/\pi})^{1/2}$$

Example: Let $M_{N+1} = 10$, $\Delta = 0.125$, $k = 0.01$,

$M_* = 15$. Calculate Q .

$$E(m) = (1 - 0.01) \ln 10 = 2.280$$

$$G(m) = .10 \cdot 2.280 \cdot (1 + 0.125) = 0.2565$$

$$z_* = (15 - 2.280) / 0.2565 = 1.6688$$

$$\Phi(z_*) \doteq 0.9556$$

$$Q(M_N, M_{N+1}) = 1 - \Phi(z_*) = .0444$$

Let V , the value of the drying charge and s , the fraction of salvageable value of the defective wood pieces be given as $V = 18775$ and $s = 0.50$.

Then, the loss $L(Q)$ can be calculated as

$$\begin{aligned} L(Q) &= V(1 - s)Q = \$ 18775 \cdot 0.50 \cdot 0.0444 \\ &= \$ 416.8 \end{aligned}$$

2.2 Shrinkage defects .

An equation relating S , a measure of deformation, and wet-bulb depression has been formulated.

$$S_{t+1} = 1.45 e^{0.033 D_t}$$

This equation represents only a gross approximation of the actual behavior of deformation during drying. A more accurate formulation would consider the wet-bulb depression for more than one interval. The normal distribution was found to adequately describe the variability of "bow-tie" type deformation in the case of pencil-slats (Rensi, 1976). The above equation can be used to estimate the mean shrinkage, $E(S)$. Given an adequate estimate of the standard deviation, $G(S)$, and a tolerance limit on maximum deformation, S_* , one can calculate R , the estimated fraction of substandard pieces.

Given R^* , the maximum acceptable level of R , a corresponding upper bound on D , say D^* , can be calculated. D^* represents the maximum allowable value of the wet-bulb depression compatible with a deformation incidence less than R^* .

Section 3. Drying costs. .

These results have been taken from Claxton (1968).

a. Evaporation

$$F_1 = W \cdot \Delta M \cdot \text{Hev}(T)$$

F_1 = heat required to evaporate the water out of the wood, in BTU's

W = weight of the wood, in lbs.

ΔM = change in moisture content, taken with positive sign

$\text{Hev}(T)$ = specific heat of evaporation at temperature T

$$\text{Hev}(T) = 1093.35 - .5723 T$$

b. Wood temperature increase

$$F_2 = W \cdot \Delta U^+ \cdot (H_{\text{wood}}(\bar{U}) + M)$$

F_2 = heat required to change the temperature of both wood and water inside the wood, in BTU's

U = wet-bulb temperature, in degrees Fahrenheit

$$U = T - D$$

$$\bar{U} = (U_{t-1} + U_t) / 2$$

$$\Delta U^+ = \begin{cases} U_t - U_{t-1} & \text{if } > 0 \\ 0 & \text{otherwise} \end{cases}$$

H_{wood} (U) = specific heat of wood at temperature U, in BTU's

$$H_{\text{wood}} (U) = .2054 + .00016 U$$

c. Kiln temperature increase

$$F_3 = \text{Air} (T) \cdot H_{\text{air}} (\text{Ratio} (U)) \cdot \Delta T^+$$

F_3 = heat required to increase the temperature in the kiln atmosphere, in BTU's

Air (T) = weight of dry air at temperature T, in lbs

$$\text{Air} (T) = A / (11.58 - 0.02525 T)$$

A = volume of air, in cubic feet

Ratio (U) = weight of water vapor per unit weight of dry air at wet-bulb temperature U

$$\text{Ratio} (U) = 0.021 + 0.0238 (U/100)^{4.2926}$$

H_{air} (R) = specific heat of wet air at Ratio (U) = R, in BTU's

$$H_{\text{air}} (R) = .241 + .444 R$$

$$\Delta T^+ = \text{Max} (0, \Delta T)$$

$$\Delta T^- = \text{Min} (0, \Delta T)$$

d. Humidity control

$$F_4 = F_{41} + F_{42} + F_{43}$$

F_4 = heat, measured in BTU's, required to bring up the temperature of the air taken in from outside in order to control the humidity

$$F_{41} = \Delta H^+ (T - T_0 + H_{\text{ev}}(T))$$

T_0 = outside temperature, in degrees Fahrenheit

U_0 = initial wet bulb temperature in the kiln,
in degrees Fahrenheit

D_0 = initial depression:

$$D_0 = T_0 - U_0$$

ΔH = change in humidity of kiln air, in lbs

ΔH^- = Min (ΔH , 0)

ΔH^+ = Max (ΔH , 0)

F_{42} = $.241 (T - T_0) \cdot (-\Delta H^-) / \text{Ratio } (U_{t-1})$

F_{43} = $.241 (T - T_0) \cdot \Delta M \cdot W / \text{Ratio } (U)$

ΔH = Air (T) $\cdot$ Ratio (U) - Air (T_{t-1}) $\cdot$ Ratio (U_{t-1})
- Vent (ΔT^-)

Vent (ΔT^-) = humidity, measured in lbs, change resulting
from venting out hot air in order to reduce
temperature by T

Vent (ΔT^-) = ΔT^- Air (T_{t-1}) $\cdot$ (Ratio (U_{t-1}) - Ratio (U_0))
 $\cdot$ H_{wair} (Ratio (U_{t-1})) / $.241 (T_{t-1} - T_0)$
+ ($.444 T_{t-1} \cdot \text{Ratio } (U_{t-1}) - T_0 \text{ Ratio } (U_0)$)